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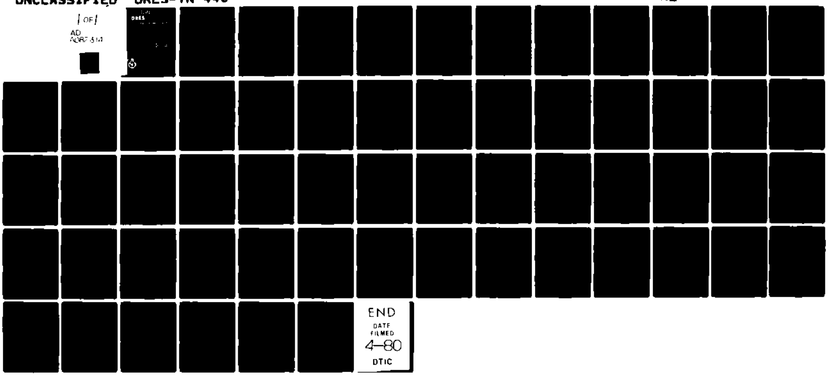

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by

D.A. Bayly

PCN 21K01

February 1980

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ABSTRACT

A cantilever beam force transducer was modelled as a massless elastic section at the base with the remaining section of the beam rigid and having mass. Computer programs were written to simulate free or forced, damped or undamped vibrations of the beam. Good agreement was found between predicted and experimental frequencies of undamped free vibration for two different beams. After further verification, the computer programs can be used to determine beam configurations, viscous damping factors, and loading rates which will reduce unwanted oscillations of the transducer element.

(U)

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DYNAMIC SIMULATION OF CANTILEVER BEAM TYPE FORCE TRANSDUCER (U)

by

D.A. Bayly

1. INTRODUCTION

The Chemistry Section at the Defence Research Establishment Suffield is studying the viscoelastic properties of fluids. It was noticed that a pencil point quickly lifted from a fluid surface would pull up a filament the size of which was an indication of the fluid's viscoelastic properties. Based on that observation, two members of the technical staff at DRES built the measurement system shown schematically in Figure 1.

The pencil point was replaced by a short length of .055-inch diameter steel rod called the T-piece. Instead of lifting the T-piece from the fluid surface, the fluid contained in a 1-inch diameter by 0.1-inch deep cup was lowered away from the T-piece. The cup rested upon the plunger of a syringe. Rate of descent of the plunger was controlled by the metering valve and vacuum pump.

The weight of the fluid filament applied a vertical load to the tip of the cantilever beam causing strain in the beam which in turn caused a change in electrical resistance of the strain gauges proportional to filament weight. As the filament would reach a maximum weight and then begin to drip back into the cup, so the strain versus time signal would show a sharp rise followed by a gradual decline. However the loading function excited oscillations of the beam which tended to obscure the desired signal.

Several remedies were considered. A small amount of grease placed between the beam and one of its support members was an effective damper, but the grease slowly oozed out and it was difficult to maintain the same degree of damping from day to day. First attempts at electronically filtering the signal had results similar to mechanical overdamping: the unwanted oscillations were removed, but so was most of the signal. A Mark 1 beam and a Mark 2 beam of slightly different dimensions were tried. While both had an oscillation problem, the Mark 2 beam vibrated at a higher frequency and with greater amplitude. Hence an analysis was attempted in order to determine what beam dimensions would minimize the oscillation problem.

The author's approach was to predict the behaviour of beam models with the IBM Continuous System Modeling Program (CSMP). Figure 2 shows the Mark 1 beam and its model. Elastic deformation was assumed to occur in the section of beam closest to the support. The mass of this section was neglected because deflections, accelerations and hence inertia forces were low compared to those of the other part of the beam. The flanged section of beam was assumed rigid and its mass significant because of relatively large deflections and inertia forces. In calculating the mass of the rigid section, the masses of any attached damper, the T-piece and fluid filament were neglected. Some compensation for those assumptions was made by ignoring the absence of mass in lightening holes. Table I defines the beam nomenclature, and Figure 3 illustrates the coordinate system. Because the position of the rigid section could be specified by two coordinates, the model was a two-degree-of-freedom system and two natural frequencies of vibration were expected.

This report describes development of a CSMP "Vibrating Cantilever" program and an auxiliary FORTRAN program. Instructions for their use are given. Users unfamiliar with CSMP will find additional background information in Reference 1. Free or forced vibration, with or without damping, can be examined. Good agreement was found between predicted and experimental frequencies of undamped free vibration for the Mark 1 and Mark 2 beams. Further verification of the program is suggested in Section 7. The most desirable beam configuration, viscous damping factor, and loading rate can then be found by examining the properties of various candidate configurations.

TABLE I

BEAM NOMENCLATURE

ℓ_1 = length of elastic portion of beam, in

ℓ_2 = length of clamp to centre of gravity of rigid portion, in

ℓ_3 = length of clamp to line of action of damping force, in

ℓ_4 = length of clamp to line of action of forcing function, in

b_1 = width of elastic portion, in

b_2 = width of rigid portion, in

d_1 = thickness of elastic portion, in

d_2 = thickness of rigid portion, in

W = specific weight of rigid material, $lb_f\ in^{-3}$

E = Young's modulus of elastic material, $lb_f\ in^{-2}$

2. DERIVATION OF EQUATIONS OF MOTION

Free body diagrams of the elastic and rigid portions of the beam are given in Figure 4. It can be shown that an elastic cantilever subjected to a vertical load P and bending moment M at its tip will have at the tip a vertical displacement

$$y_1 = \frac{P\ell_1{}^3}{3EI} + \frac{M\ell_1{}^2}{2EI} \qquad (1)$$

and an angular displacement

$$\theta = \frac{M\ell_1}{EI} + \frac{P\ell_1^2}{2EI} \tag{2}$$

where I is the second moment of area of the cross-section of the elastic portion of the beam.

$$I = \frac{b_1 d_1^3}{12} \quad in^4 . \tag{3}$$

Summing the vertical forces acting on the rigid portion,

$$F_F - P - F_D = m\ddot{y}_2 \tag{4}$$

and summing moments

$$P(\ell_2-\ell_1)-F_D(\ell_3-\ell_2)+F_F(\ell_4-\ell_2)-M = J\ddot{\theta} \tag{5}$$

where J is the second moment of mass of the rigid portion about a transverse axis through its centroid. The mass of the rigid portion is m, in units of $lb_f\ s^2in^{-1}$.

$$m = W(\ell_4-\ell_1)b_2d_2 \times \frac{1}{32.2x12} \quad lb_f\ s^2in^{-1} \tag{6}$$

$$J = \frac{m}{12}\left[(\ell_4-\ell_1)^2 + d_2^2\right] lb_f\ s^2\ in \tag{7}$$

For small vibrations $y_2 = y_1 + (\ell_2-\ell_1)\theta$ (8)

$$\dot{y}_2 = \dot{y}_1 + (\ell_2-\ell_1)\dot{\theta} \tag{9}$$

$$\ddot{y}_2 = \ddot{y}_1 + (\ell_2-\ell_1)\ddot{\theta} \tag{10}$$

(4) becomes $P = F_F - F_D - m[\ddot{y}_1 + (\ell_2 - \ell_1)\ddot{\theta}]$ (11)

(5) becomes $M = \{F_F - F_D - m[\ddot{y}_1 + (\ell_2 - \ell_1)\ddot{\theta}]\} \quad (\ell_2 - \ell_1)$

$$-F_D(\ell_3 - \ell_2) + F_F(\ell_4 - \ell_2) - J\ddot{\theta}$$

$$M = F_F(\ell_2 - \ell_1 + \ell_4 - \ell_2) + F_D(\ell_1 - \ell_2 - \ell_3 + \ell_2)$$

$$-m[\ddot{y}_1 + (\ell_2 - \ell_1)\ddot{\theta}](\ell_2 - \ell_1) - J\ddot{\theta}$$

$$M = F_F(\ell_4 - \ell_1) - F_D(\ell_3 - \ell_1) - m[\ddot{y}_1 + (\ell_2 - \ell_1)\ddot{\theta}](\ell_2 - \ell_1) - J\ddot{\theta} \quad . \tag{12}$$

Substituting for P and M in (1)

$$y_1 = \frac{F_F\ \ell_1{}^3}{3EI} - \frac{F_D\ \ell_1{}^3}{3EI} - \frac{m\ell_1{}^3}{3EI}[\ddot{y}_1 + (\ell_2 - \ell_1)\ddot{\theta}]$$

$$+ \frac{F_F(\ell_4 - \ell_1)\ell_1{}^2}{2EI} - \frac{F_D(\ell_3 - \ell_1)\ell_1{}^2}{2EI} - \frac{m\ell_1{}^2}{2EI}[\ddot{y}_1$$

$$+ (\ell_2 - \ell_1)\ddot{\theta}](\ell_2 - \ell_1) - \frac{\ell_1{}^2}{2EI} J\ddot{\theta}$$

$$y_1 = F_F\left[\frac{\ell_1{}^3}{3EI} + \frac{(\ell_4 - \ell_1)\ell_1{}^2}{2EI}\right]$$

$$- F_D\left[\frac{\ell_1{}^3}{3EI} + \frac{(\ell_3 - \ell_1)\ell_1{}^2}{2EI}\right]$$

$$+ \ddot{y}_1\left[\frac{-m\ell_1{}^3}{3EI} - \frac{m\ell_1{}^2}{2EI}(\ell_2 - \ell_1)\right]$$

$$+ \ddot{\theta}\left[\frac{-m\ell_1{}^3}{3EI}(\ell_2 - \ell_1) - \frac{m\ell_1{}^2}{2EI}(\ell_2 - \ell_1)^2 - \frac{\ell_1{}^2}{2EI} J\right]$$

$$y_1 = F_F\left[\frac{\ell_1{}^2}{2EI}(\ell_4 - \frac{\ell_1}{3})\right]$$

$$- F_D\left[\frac{\ell_1{}^2}{2EI}(\ell_3 - \frac{\ell_1}{3})\right]$$

$$+ \ddot{y}_1\left[\frac{-m\ell_1{}^2}{2EI}(\ell_2 - \frac{\ell_1}{3})\right]$$

$$+ \ddot{\theta}\left\{\frac{-\ell_1{}^2}{EI}\left[m\left(\frac{\ell_1{}^2}{6} - \frac{2}{3}\ell_1\ell_2 + \frac{\ell_2{}^2}{2}\right) + \frac{J}{2}\right]\right\} .$$

Define $G = \frac{\ell_1{}^2}{2EI}(\ell_4 - \frac{\ell_1}{3})$ (13)

$$H = \frac{\ell_1{}^2}{2EI}(\ell_3 - \frac{\ell_1}{3}) \quad (14)$$

$$A = \frac{-m\ell_1{}^2}{2EI}(\ell_2 - \frac{\ell_1}{3}) \quad (15)$$

$$B = \frac{-\ell_1{}^2}{EI}\left[m\left(\frac{\ell_1{}^2}{6} - \frac{2}{3}\ell_1\ell_2 + \frac{\ell_2{}^2}{2}\right) + \frac{J}{2}\right]. \quad (16)$$

Then $y_1 = G\,F_F - H\,F_D + A\ddot{y}_1 + B\ddot{\theta}$. (17)

Substituting for P and M in (2)

$$\theta = F_F(\ell_4-\ell_1)\frac{\ell_1}{EI} - F_D(\ell_3-\ell_1)\frac{\ell_1}{EI} - \frac{m\ell_1}{EI}[\ddot{y}_1 + (\ell_2-\ell_1)\ddot{\theta}](\ell_2-\ell_1)$$

$$- \frac{J\,\ell_1}{EI}\ddot{\theta} + \frac{F_F\,\ell_1{}^2}{2EI} - \frac{F_D\,\ell_1{}^2}{2EI} - \frac{m\ell_1{}^2}{2EI}[\ddot{y}_1 + (\ell_2-\ell_1)\ddot{\theta}]$$

$$\theta = F_F\left(\frac{\ell_1}{EI}\right)\left(\ell_4 - \ell_1 + \frac{\ell_1}{2}\right)$$
$$- F_D\left(\frac{\ell_1}{EI}\right)\left(\ell_3 - \ell_1 + \frac{\ell_1}{2}\right)$$
$$+ \ddot{y}_1\left(\frac{-m\ell_1}{EI}\right)\left(\ell_2 - \ell_1 + \frac{\ell_1}{2}\right)$$
$$+ \ddot{\theta}\left(\frac{\ell_1}{EI}\right)\left[-m(\ell_2-\ell_1)^2 - J - \frac{m\ell_1}{2}(\ell_2-\ell_1)\right]$$

$$\theta = F_F\left[\frac{\ell_1}{EI}\left(\ell_4 - \frac{\ell_1}{2}\right)\right]$$
$$- F_D\left[\frac{\ell_1}{EI}\left(\ell_3 - \frac{\ell_1}{2}\right)\right]$$
$$+ \ddot{y}_1\left[\frac{-m\ell_1}{EI}\left(\ell_2 - \frac{\ell_1}{2}\right)\right]$$
$$+ \ddot{\theta}\left\{\frac{\ell_1}{EI}\left[-m\left(\ell_2^2 - \frac{3}{2}\ell_1\ell_2 + \frac{\ell_1^2}{2}\right) - J\right]\right\} .$$

Define $$K = \frac{\ell_1}{EI}\left(\ell_4 - \frac{\ell_1}{2}\right) \tag{18}$$

$$L = \frac{\ell_1}{EI}\left(\ell_3 - \frac{\ell_1}{2}\right) \tag{19}$$

$$C = \frac{-m\ell_1}{EI}\left(\ell_2 - \frac{\ell_1}{2}\right) \tag{20}$$

$$D = \frac{\ell_1}{EI}\left[-m\left(\ell_2^2 - \frac{3}{2}\ell_1\ell_2 + \frac{\ell_1^2}{2}\right) - J\right] . \tag{21}$$

Then $\theta = K\,F_F - L\,F_D + C\ddot{y}_1 + D\ddot{\theta}$ (22)

(17) and (22) are sufficient to describe the motion of the beam.

3. CSMP PROGRAM

CSMP, the Continuous System Modeling Program, is an IBM package which performs a digital simulation of an analog computer (Reference 1). The user writes differential equations to describe the process being simulated, and inputs initial conditions, constants and other relevant information to the program in a specified format. CSMP can list five variables as functions of time, and can plot one variable as a function of any other variable. Run time for the CSMP "Vibrating Cantilever" program is typically five minutes, after which the user can change initial conditions or constants through the 1130 keyboard. The program demonstrates the effects of changes in dimensions, material density and stiffness, viscous damping factor, and added (or subtracted) mass upon the natural vibration frequencies of the elastic-rigid cantilever beam. It also demonstrates the effect of those changes upon the transient response of the beam to any tip-applied vertical forcing function.

3.1 Equations of Motion

The equations of motion, from section 2, are:

$$y_1 = G\,F_F - H\,F_D + A\ddot{y}_1 + B\ddot{\theta} \qquad (17)$$

$$\theta = K\,F_F - L\,F_D + C\ddot{y}_1 + D\ddot{\theta} \qquad (22)$$

In order to facilitate construction of the CSMP block diagram, the equations are manipulated as follows:

From (17) $$\ddot{y}_1 = \frac{y_1 - GF_F + H\,F_D - B\ddot{\theta}}{A}\,. \qquad (23)$$

From (22) $$\ddot{y}_1 = \frac{\theta - K\,F_F + L\,F_D - D\ddot{\theta}}{C}\,. \qquad (24)$$

Therefore $Cy_1 - CGF_F + CHF_D - CB\ddot{\theta}$

$$= A\theta - KAF_F + LAF_D - AD\ddot{\theta}$$

$$\ddot{\theta}(CB-AD) = -A\theta + (KA-CG)F_F + Cy_1 + (CH-LA)F_D$$

$$\ddot{\theta} = \frac{-A}{CB-AD}\theta + \frac{C}{CB-AD}\ y_1 + \left(\frac{KA-CG}{CB-AD}\right) F_F + \left(\frac{CH-LA}{CB-AD}\right) F_D \ . \qquad (25)$$

Rewriting (24),

$$\ddot{y}_1 = \frac{1}{C}\theta - \frac{D}{C}\ddot{\theta} - \frac{K}{C}F_F + \frac{L}{C}F_D \quad . \qquad (26)$$

(25) and (26) are the equations of motion presented by the portion of the CSMP block diagram shown in Figure 5.

3.2 Viscous Damping

$$F_D = N\dot{y}_3$$

$$= N[\dot{y}_1 + (\ell_3-\ell_1)\dot{\theta}]. \qquad (27)$$

This portion of the CSMP block diagram is shown in Figure 6.

N is the damping factor with units of lb_f s in^{-1}. It is set equal to zero for undamped vibration, or can take on any positive value to simulate a desired degree of viscous damping. The damper shown in Figure 7 was assumed to exist in order to estimate a typical magnitude of the damping factor.

It was assumed that there was a linear velocity distribution in the oil between the plate, moving with velocity $\dot{y}_3\ \frac{in}{s}$, and the stationary wall. Thus the velocity gradient was $\frac{d\dot{y}}{dx} = \frac{\dot{y}_3}{.058}\ s^{-1}$. According to Newton's law of viscosity

$$\tau = \mu \frac{d\dot{y}}{dx}$$

where τ = shear stress, $lb_f\ in^{-2}$

μ = viscosity of the oil, $lb_f\ s\ in^{-2}$

= $.00142\ lb_f\ s\ in^{-2}$.

The total area of the plate acted upon by the shear stress was

$$A_{PLATE} = 2 \times .25 \times .25 = .125\ in^2.$$

The damping force was

$$F_D = \tau A_{PLATE}$$

$$= .00142 \times \frac{\dot{y}_3}{.058} \times .125$$

$$= .00306 \times \dot{y}_3 \quad .$$

From (27), $N = \dfrac{F_D}{\dot{y}_3}$

$$= .00306\ lb_f\ s\ in^{-1} \ .$$

3.3 Forcing Function

The force applied to the tip of the beam, F_F, was modelled as a linear rise to the peak force, F_{FMAX}, at time t = .1T, followed by an exponential decay. Figure 8 illustrates the nondimensionalized forcing function F_F/F_{FMAX} versus t/T.

From t/T = 0 to t/T=.1,

$$\frac{F_F}{F_{FMAX}} = 10\,\frac{t}{T} \quad .$$

From t/T = .1 to t/T 1.0,

$$\frac{F_F}{F_{FMAX}} = e^{-k\left(\frac{t}{T} - .1\right)}.$$

To evaluate the constant k it was assumed that

$$F_F/F_{FMAX} = .2 \text{ when } t/T = .6$$

$$.2 = e^{-k(.6-.1)}$$

$$= e^{-.5k}$$

$$k = \frac{\ell n(.2)}{-.5}$$

$$= 3.219 \quad .$$

Therefore $$\frac{F_F}{F_{FMAX}} = e^{-3.219\left(\frac{t}{T} - .1\right)} \quad , .1 \le \frac{t}{T} \le 1.0 \quad .$$

Table II lists values of F_F/F_{FMAX} at regular intervals of t/T.

TABLE II

$$\frac{F_F}{F_{FMAX}} \text{ vs } \frac{t}{T}$$

$\frac{t}{T}$	$\frac{F_F}{F_{FMAX}}$
0.	0.
.1	1.0
.2	.7246
.3	.5252
.4	.3806
.5	.2758
.6	.2000
.7	.1448
.8	.1050
.9	.0760
1.0	.0552

The forcing function part of the CSMP block diagram is shown in Figure 9. In the CSMP program, the values of Table 2 are read in as specifications for a Function Generator Block, Block 17. The times of the beginning and end of the force pulse, i.e., 0 s and T s, must also be specified as Parameters 2 and 1, respectively, of the Function Generator Block. Input to the Function Generator is t from the time base, Block 76. Output is F_F/F_{FMAX}. When t/T is not a multiple of .1, the Function Generator linearly interpolates a value for F_F/F_{FMAX}. For t/T greater than 1.0, F_F/F_{FMAX} is constant at .0552. F_{FMAX} is specified as Parameter 1 of a Gain Block, Block 18, which receives F_F/F_{FMAX} as input, and which outputs F_F.

3.4 Strain

It was desired that the CSMP program should continuously calculate the maximum strain at the base of the beam, because strain gauges were placed there on the actual beam. Linear elastic deformation was assumed, and the dead weight of the beam was neglected.

$$\varepsilon = \frac{\sigma}{E} \tag{28}$$

where ε = strain, in in^{-1}

σ = stress, $lb_f\ in^{-2}$

E = Young's modulus of beam material, $lb_f\ in^{-2}$.

Applying the basic flexure formula,

$$\frac{\sigma}{E} = \frac{1}{E} \times \frac{M_0\, d_1}{2I} \tag{29}$$

where M_0 = bending moment at base of beam, lb_f in

d_1 = thickness of beam at its base, in

I = second moment of area of beam cross-section at its base, in^4. See equation (3).

From the free body diagram, Figure 4, it can be shown that

$$M_0 = P\ell_1 + M\ . \tag{30}$$

From equation (5),

$$M = P(\ell_2-\ell_1) - F_D(\ell_3-\ell_2) + F_F(\ell_4-\ell_2) - J\ddot{\theta} \ . \tag{31}$$

Recalling equation (11),

$$P = F_F - F_D - m[\ddot{y}_1 + (\ell_2-\ell_1)\ddot{\theta}] \ . \tag{11}$$

Equating the left-hand side of (29) to the right-hand side of (30) and substituting for M_0, P and M,

$$\varepsilon = \frac{d_1}{2EI}\left[\ell_1\left\{F_F - F_D - m[\ddot{y}_1+(\ell_2-\ell_1)\ddot{\theta}]\right\} + P(\ell_2-\ell_1) - F_D(\ell_3-\ell_2) - F_F(\ell_4-\ell_2) - J\ddot{\theta}\right] \ . \tag{32}$$

Equation (32) is represented by the portion of the CSMP block diagram shown in Figure 10.

Figure 11 shows the complete CSMP block diagram. Appendix A is a listing of the CSMP program for a typical run.

4. FORTRAN PROGRAM

The main purpose of the FORTRAN program is to punch Initial Condition/Parameter cards for the CSMP program. It also prints a listing of the parameters. Only the parameters dependent on beam dimensions are calculated by this program.

Figure 12 shows a flow chart of the FORTRAN program. It begins by reading a beam dimension data card. Parameters needed for the CSMP simulation of this particular beam are calculated and stored in an output matrix. The program reads another card, calculates and stores the next set of parameters, and so on until a blank card is read. Then the parameter listing is printed and the Initial Condition/Parameter cards for the CSMP program are punched.

Table III defines the variables, and Appendix B is a listing of the program with a sample of output.

The variable ADMAS represents mass added to the beam at the centre of gravity of the rigid portion. A nut and bolt or a lump of solder might be attached to the beam to lower the natural frequencies of vibration. The programs are only valid for mass added at the C of G of the rigid portion because they neglect any change in the second moment of mass, J, due to the added mass.

TABLE III

FORTRAN VARIABLES

Variable	Definition	Mode	Class
U(K,N)	beam dimension input matrix	Real	2
V(L,N)	parameter output matrix	"	2
EMM(N)	beam mass, g	"	1
N	index to distinguish data sets	Integer	0
EL1	ℓ_1, in	Real	0
EL2	ℓ_2, in	"	0
EL3	ℓ_3, in	"	0
EL4	ℓ_4, in	"	0
B1	b_1, in	"	0
B2	b_2, in	"	0
D1	d_1, in	"	0
D2	d_2, in	"	0
W	w, $lb_f\ in^{-3}$	"	0
E	E, $lb_f\ in^{-2}$	"	0
ADMAS	mass added to C of G of rigid portion of beam, g	"	0
EM	m, $lb_f\ s^2\ in^{-1}$. See equation (6)	"	0
REALJ	J, $lb_f\ s^2\ in$. See equation (7)	"	0
REALI	I, in^4. See equation (3)	"	0
A	A, s^2. See equation (15)	"	0
B	B, in s^2. See equation (16)	"	0

TABLE III (continued)

Variable	Definition	Mode	Class
C	C, $in^{-1}\ s^2$. See equation (20)	"	0
D	D, s^2. See equation (21)	"	0
G	G, in lb_f^{-1}. See equation (13)	"	0
H	H, in lb_f^{-1}. See equation (14)	"	0
REALK	K, lb_f^{-1}. See equation (18)	"	0
REALL	L, lb_f^{-1}. See equation (19)	"	0
P1B3	parameter 1, block 3, s^2	Real	0
P1B4	parameter 1, block 4, in	"	0
P2B5	parameter 2, block 5, s^2	"	0
P1B6	parameter 1, block 6, in s^{-2}	"	0
P2B6	parameter 2, block 6, lb_f^{-1} in s^{-2}	"	0
P3B6	parameter 3, block 6, lb_f^{-1} in s^{-2}	"	0
P1B7	parameter 1, block 7, $lb_f\ s^2\ in^{-1}$	"	0
P1B10	parameter 1, block 10, in	"	0
P2B10	parameter 2, block 10, in	"	0
P3B10	parameter 3, block 10, in	"	0
P1B11	parameter 1, block 11, in	"	0
P1B12	parameter 1, block 12, $in^{-1}\ lb_f^{-1}$	"	0
P2B13	parameter 2, block 13, in	"	0
P1B15	parameter 1, block 15, $in^{-1}\ s^{-2}$	"	0
P2B15	parameter 2, block 15, $lb_f^{-1}\ s^{-2}$	"	0
P3B15	parameter 3, block 15, $lb_f^{-1}\ s^{-2}$	"	0
P2B16	parameter 2, block 16, $lb_f\ s^2$ in	"	0
J	total number of data sets	Integer	0

5. USE OF PROGRAMS

5.1 CSMP

Reference 1 gives a detailed description of how to run a CSMP program. Only a brief outline is given here.

1. Prepare a card deck which includes appropriate control cards: Configuration Specification cards, Initial Condition/Parameter Specification cards, Function Generator Specification cards and about a dozen blank cards.
2. Ensure that the correct disc is installed.
3. Load the card deck and start the program.
4. The console printer will list the Configuration Specifications, I.C./Parameter Specifications and Function Generator Specifications. It will list the console switches to be operated, after a run, when choosing program options.
5. The console printer will ask the operator to enter values for:
   - Integration Interval
   - Total Time
   - Block for Y-Axis of Plot
   - Minimum and Maximum Values on Y-Axis
   - Block for X-Axis of Plot
   - Minimum and Maximum Values on X-Axis.
6. Press the "Start" button on the console and the plotter will draw the plot frame.
7. The console printer will ask the operator to enter:
   - Print Interval
   - Block Numbers for five variables to be listed on the line printer.

   As soon as the last Block Number is entered, the run will automatically begin.

8. The run will continue until the "Total Time" is reached, or may be stopped sooner by operating Switch 0 on the console.
9. If another run is required, operate console switches to choose program options. For example, a new plot frame will be drawn if Switch 7 is operated, and Initial Conditions/Parameters can be changed if Switch 2 is operated. Then press "Start".
10. If Configuration, I.C./Parameter or Function Generator Specifications are to be changed, the program will attempt to read new specifications from the card deck. After a blank card is read, the program will ask for input from the console.

Other specifications are changed through the console only.

11. Press "Interrupt Request" to stop the program after the last run.

Table IV shows how several runs could be set up to predict the behaviour of a given beam under various conditions.

Selection of initial y_1 and θ is arbitrary. However, a common method of observing the free vibration of an actual beam would be to deflect the tip a certain amount, y_4, then suddenly release it. Appendix C explains how to calculate initial y_1 and θ when the initial tip deflection, y_4, is specified.

Choice of Integration Interval involves a compromise between run time and the accuracy of the numerical integration process performed by the CSMP program. The best interval is the longest one for which the integration process is stable. If the interval is too long, the amplitude of the predicted vibrations increases with time even when no external force is applied, an obvious error. Cases with no damping require the shortest Integration Intervals because the frequencies of vibration are highest. A good first guess for the Integration Interval is one-tenth the period of the highest frequency vibration expected.

TABLE IV

BEHAVIOUR OF BEAM UNDER VARIOUS CONDITIONS

Purpose of Run	Parameter 1, Block 5: Initial θ	Parameter 1, Block 9: Initial y_1	Parameter 1, Block 14: N	Parameter 1, Block 18: F_{FMAX}	Y-Axis of Plot	X-Axis of Plot
To predict undamped natural frequencies	non-zero	non-zero	0	0	Block 12 (ε)	Block 76 (t)
To predict damped natural frequencies	non-zero	non-zero	positive	0	"	"
To predict transient response of undamped beam to forcing function	0	0	0	positive	"	"
To predict transient response of damped beam to forcing function	0	0	positive	positive	"	"

5.2 FORTRAN

1. Prepare a card deck as listed in Appendix B, with beam data cards in the format illustrated by Table V. This program accepts up to five beam data cards. Change the DIMENSION statement if more cards are to be processed in one run. Following the n beam data cards should be at least 11n blanks on which will be punched the CSMP I.C./Parameter Specifications.
2. Load card deck and start the program.
3. When the run is finished, transfer the newly-punched I.C./Parameter cards to the CSMP deck.

6. RESULTS

The CSMP program was used to predict frequencies of undamped free vibration for the Mark 1 and Mark 2 beams. Table VI lists some of the parameters, while Figures 13 and 14 are strain vs time plots. As expected from a two-degree-of-freedom system, there are two waveforms superimposed. Good agreement between predicted and experimental frequencies is shown in Table VI. The higher frequencies were not observed experimentally due to the resolution of the oscilloscope on which strain vs time was displayed.

A third CSMP run predicted frequencies of undamped free vibration for a Mark 1 beam with a .500 g mass added at the centre of gravity of the rigid portion. The results for this run are shown in Table VI and Figure 15.

7. CONCLUSIONS

The CSMP "Vibrating Cantilever" program made good predictions of the undamped free vibration frequencies of the Mark 1 and Mark 2 beams. It can therefore be used with confidence to make similar predictions about other elastic-rigid cantilevers.

Further verification of the program might involve the comparison

TABLE V

FORTRAN PROGRAM INPUT FORMAT

COLUMN	1–5	6–7	8–12	13–14	15–19	20–21	22–26	27–28	29–33	34–35	36–40	41–42	43–47	48–49	50–54	55–56	57–61	62–63	64–72	73–74	75–80
SAMPLE DATA	0.750		1.875		2.500		3.000		0.250		0.375		0.010		0.010		0.295		1.6100000		0.000
FORMAT	F5.3	2 X	F5.3	2 X	F5.3	2 X	F5.3	2 X	F5.3	2 X	F5.3	2 X	F5.3	2 X	F5.3	2 X	F5.3	2 X	F9.0	2 X	F5.3
VARIABLE	ℓ_1		ℓ_2		ℓ_3		ℓ_4		b_1		b_2		d_1		d_2		w		E		ADMAS
UNITS	in		in		in		in		in		in		in		in		lbf in^{-3}		lbf in^{-2}		g

of predicted to experimental strain amplitudes during undamped free vibration. The most rigorous test would be a comparison of predicted to experimental damped transient response to the forcing function. Accurate estimates of damping factor, peak force and duration of the force pulse would have to be supplied to the program.

Leaving aside the necessity for verification, the program can be used to predict the effect on unwanted oscillations of changes in:

1) beam dimensions
2) mass added or subtracted at the centre of gravity of the rigid (flanged) section
3) viscous damping factor
4) rate of loading

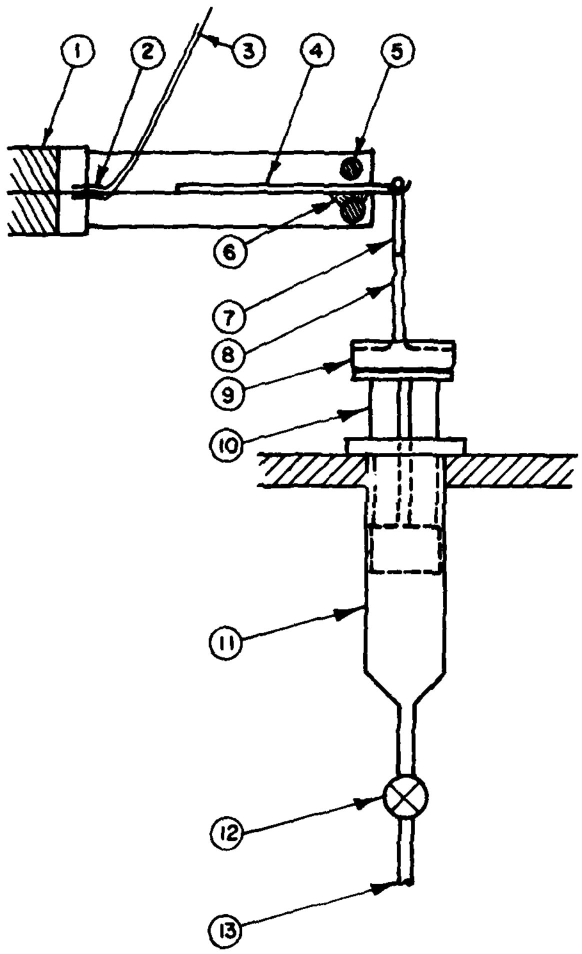
TABLE VI

UNDAMPED FREE VIBRATION

Beam Dimensions	Run 1 Mark 1 Beam	Run 2 Mark 2 Beam	Run 3 Mark 1 + .500 g Mass
Beam Dimensions			
ℓ_1 in	.750	.600	.750
ℓ_2 "	1.875	1.425	1.875
ℓ_3 "	2.500	1.750	2.500
ℓ_4 "	3.000	2.250	3.000
b_1 "	.250	0.250	.250
b_2 "	.375	.500	.375
d_1 "	.010	.020	.010
d_2 "	.010	.020	.010
Values assumed for phosphor bronze { w lb_f in^{-3}	.295	.295	.295
E lb_f in^{-2}	16100000.	16100000.	16100000.
ADMAS g	0.	0.	.500
Integration Interval s	.00005	.00001	.00005
Total Time s	.050	.050	.050
Initial y_4 in	.025	.025	.025
" y_1 in	.00343	.00525	.00343
" θ rad	.00938	.01197	.00938
N lb_f s in^{-1}	0.	0.	0.
F_{FMAX} lb_f	0.	0.	0.
Predicted Frequencies of Vibration Hz	25 550	74 1500	22 500
Experimental Frequencies of Vibration Hz	28	60 to 70	--

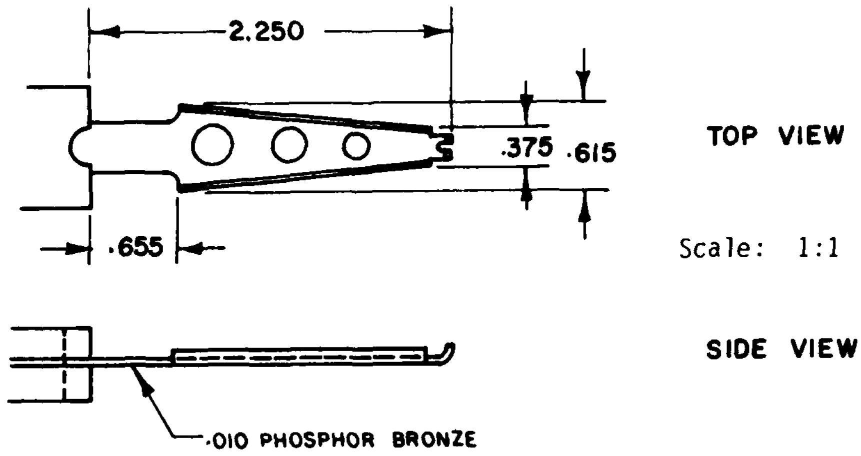
REFERENCES

1. 1130 Continuous System Modeling Program: Program Description and Operations Manual. IBM Corporation, White Plains, New York, U.S.A.

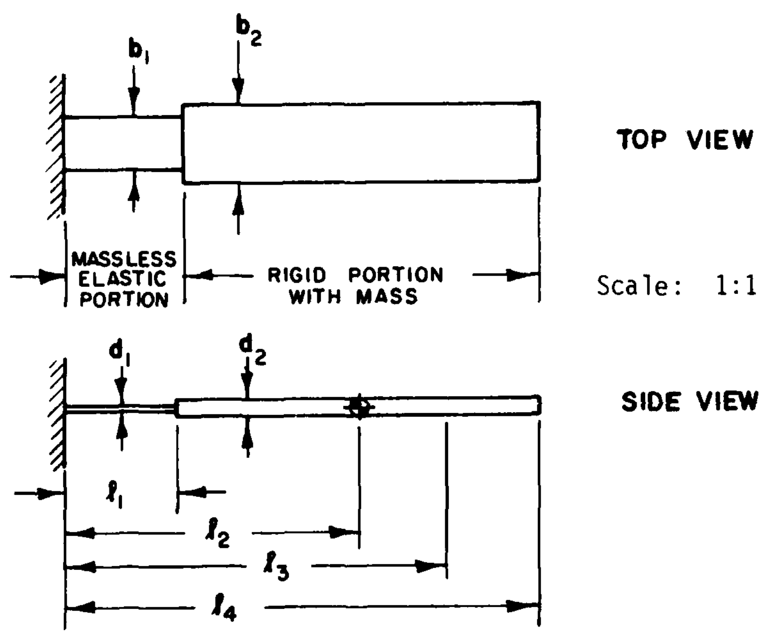


1. Beam Support Structure
2. Strain Gauges
3. Leads to Signal Conditioning Equipment and Oscilloscope
4. Beam
5. Cross Members to Limit Travel of Beam in Case of Accidental Overload
6. Blob of Silicone Grease Acting as Damper
7. T-Piece
8. Fluid Filament
9. Fluid Cup
10. Plunger
11. Syringe
12. Metering Valve
13. Tube Leading to Vacuum Pump

Figure 1: Apparatus Schematic



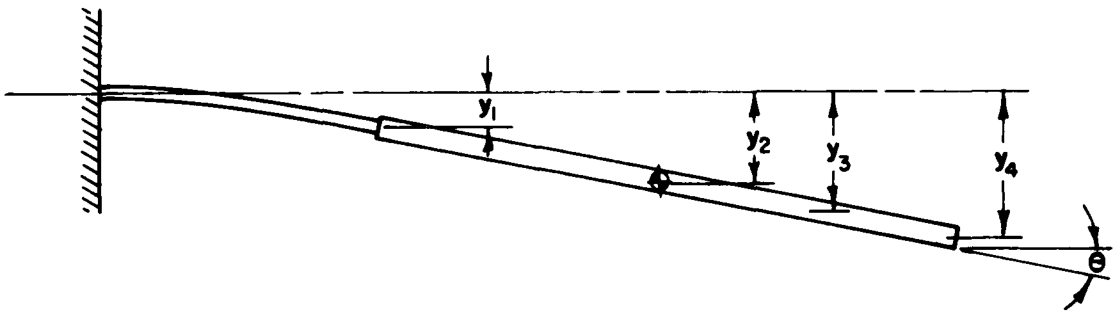
Mark 1 Beam



ℓ_1 = .750 in
ℓ_2 = 1.875 in
ℓ_3 = 2.500 in
ℓ_4 = 3.000 in
b_1 = .250 in
b_2 = .375 in
d_1 = .010 in
d_2 = .010 in

Simplified Model

Figure 2: Mark 1 Beam and Simplified Model



y_1 = Vertical displacement of elastic-rigid junction from rest position, in

y_2 = Vertical displacement of C of G of rigid position from rest position, in

y_3 = Vertical displacement of damping force point of application from rest position, in

y_4 = Vertical displacement of tip

θ = Angular displacement of rigid position of beam from rest position, rad

Figure 3: Coordinates

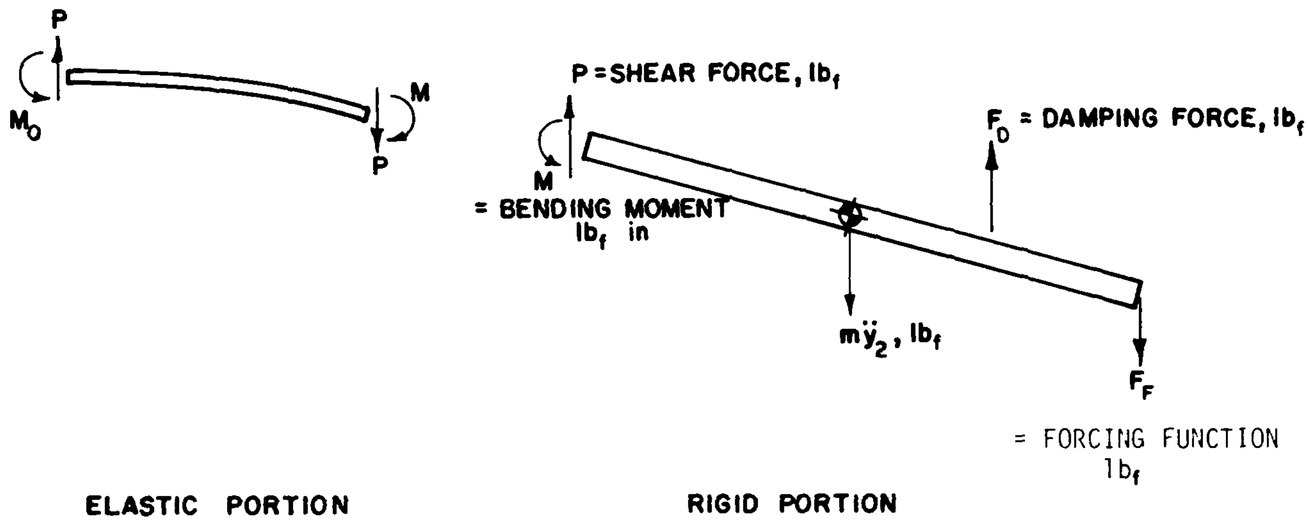
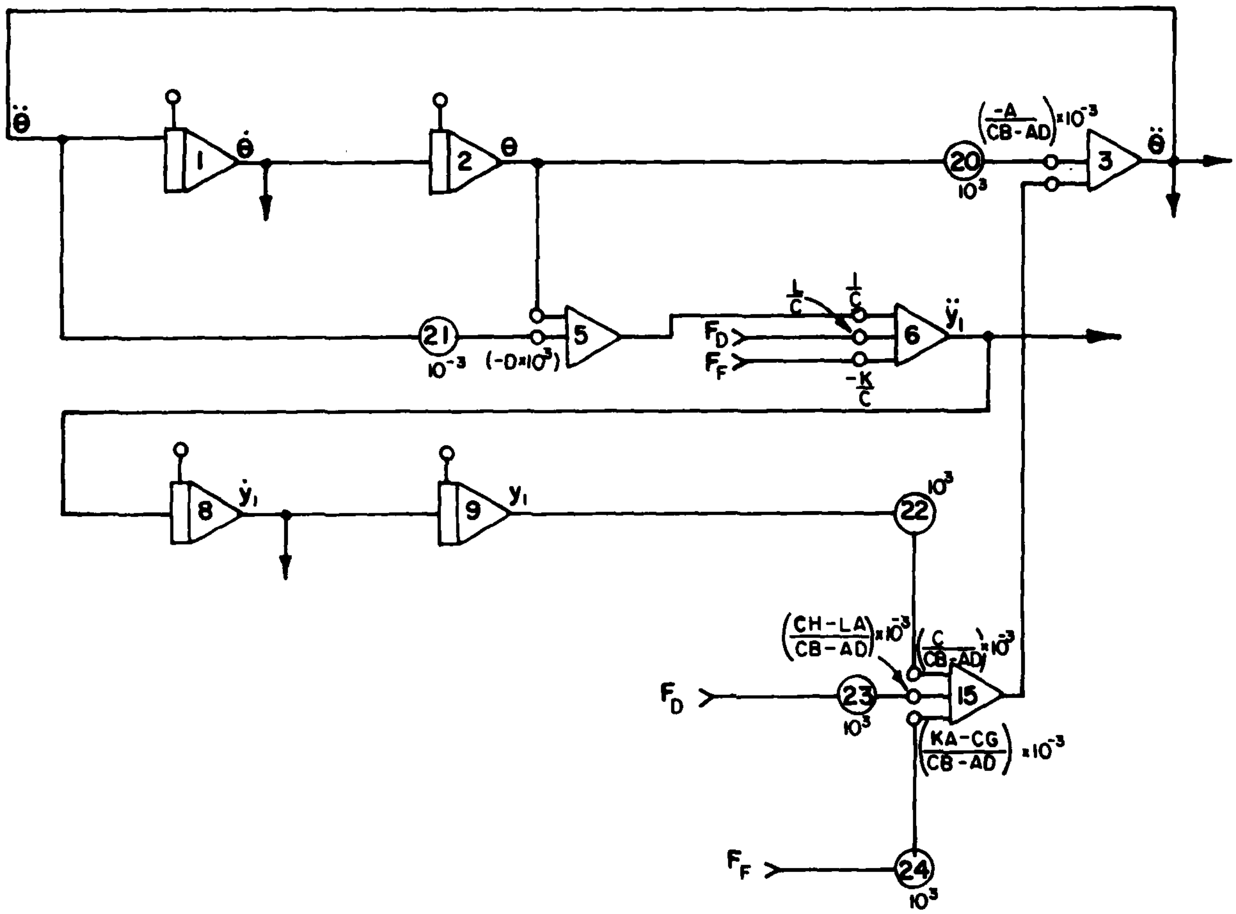


Figure 4: Free Body Diagrams



NOTES: 1. Arrowheads represent connections with other parts of the block diagram.

2. Blocks 20, 21, 22, 23 and 24 are gain factors which allow a sufficient number of significant digits when reading in the parameters for blocks 3, 5 and 15.

Figure 5: CSMP Block Diagram - Equations of Motion

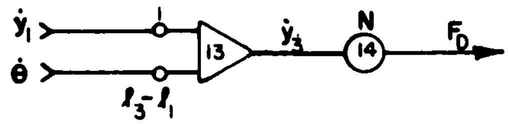
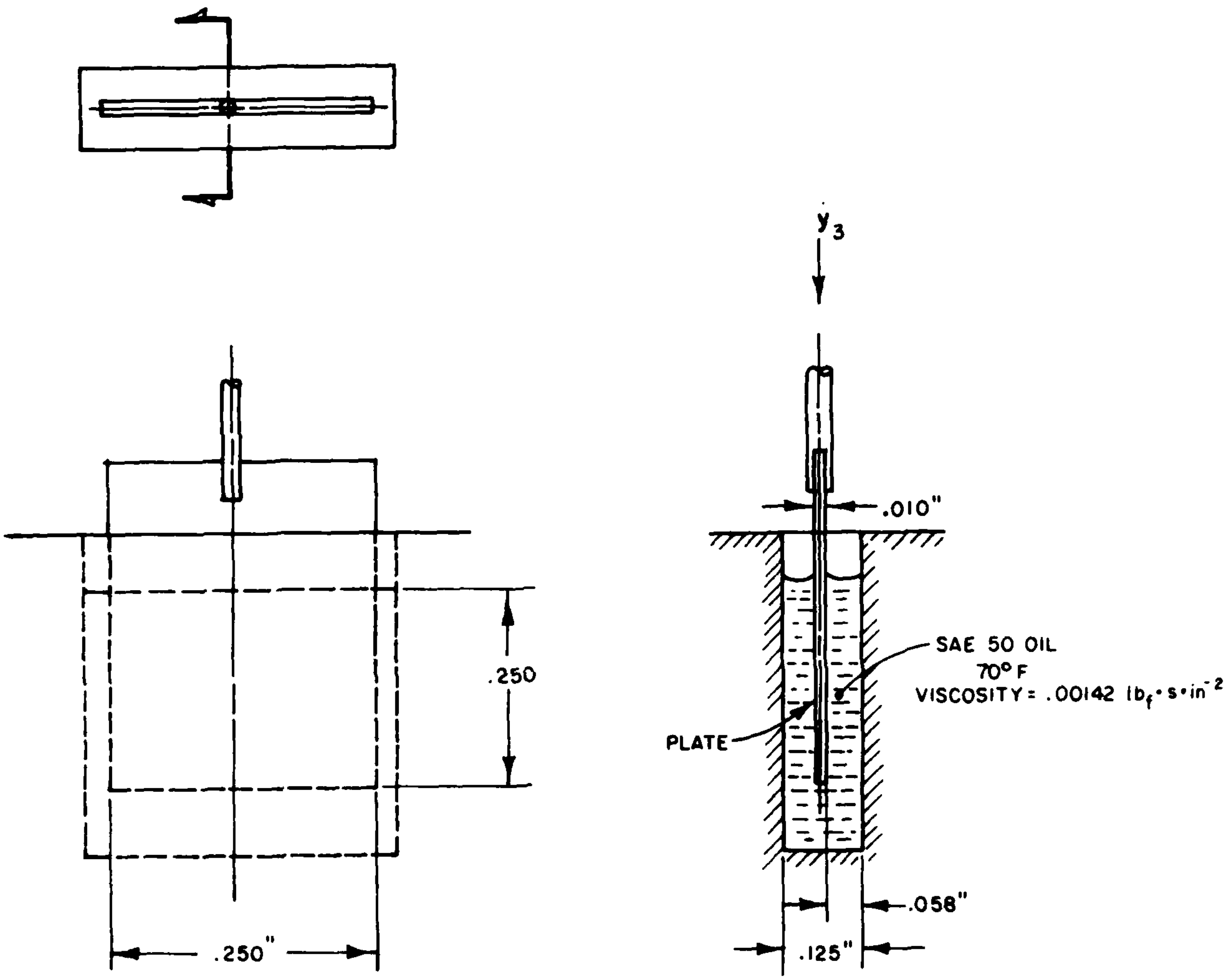


Figure 6: CSMP Block Diagram - Viscous Damping



Not to Scale

Figure 7: Quantities Used in Calculation of Typical Damping Factor

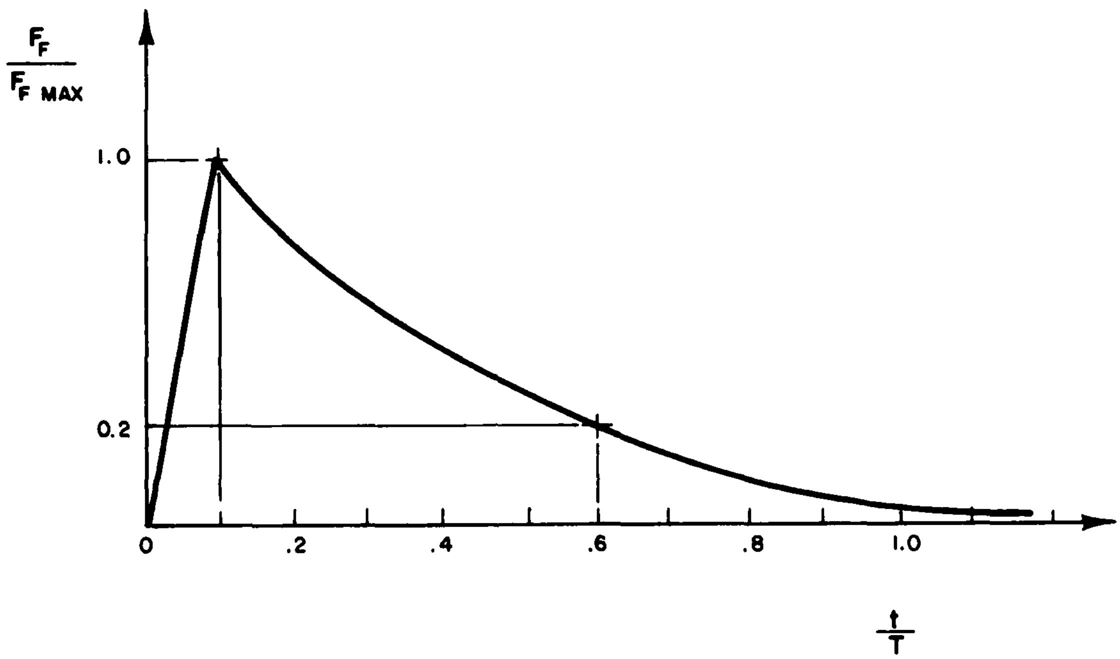


Figure 8: Non-Dimensionalized Forcing Function

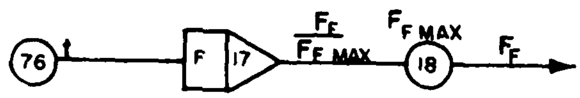
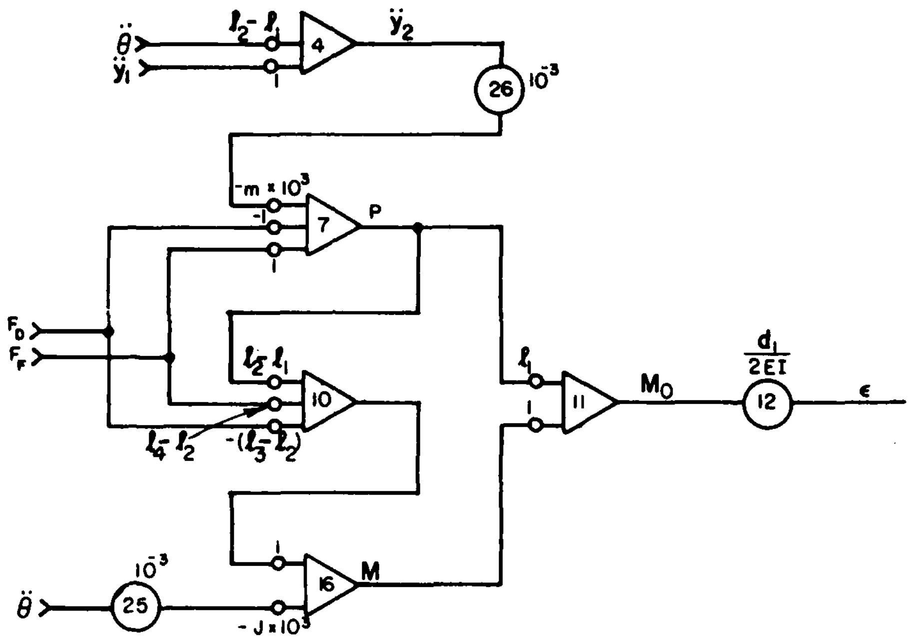


Figure 9: CSMP Block Diagram - Forcing Function



NOTE: 1. Blocks 25 and 26 are gain factors which allow a sufficient number of significant digits when reading in the parameters for blocks 16 and 7.

Figure 10: CSMP Block Diagram - Strain

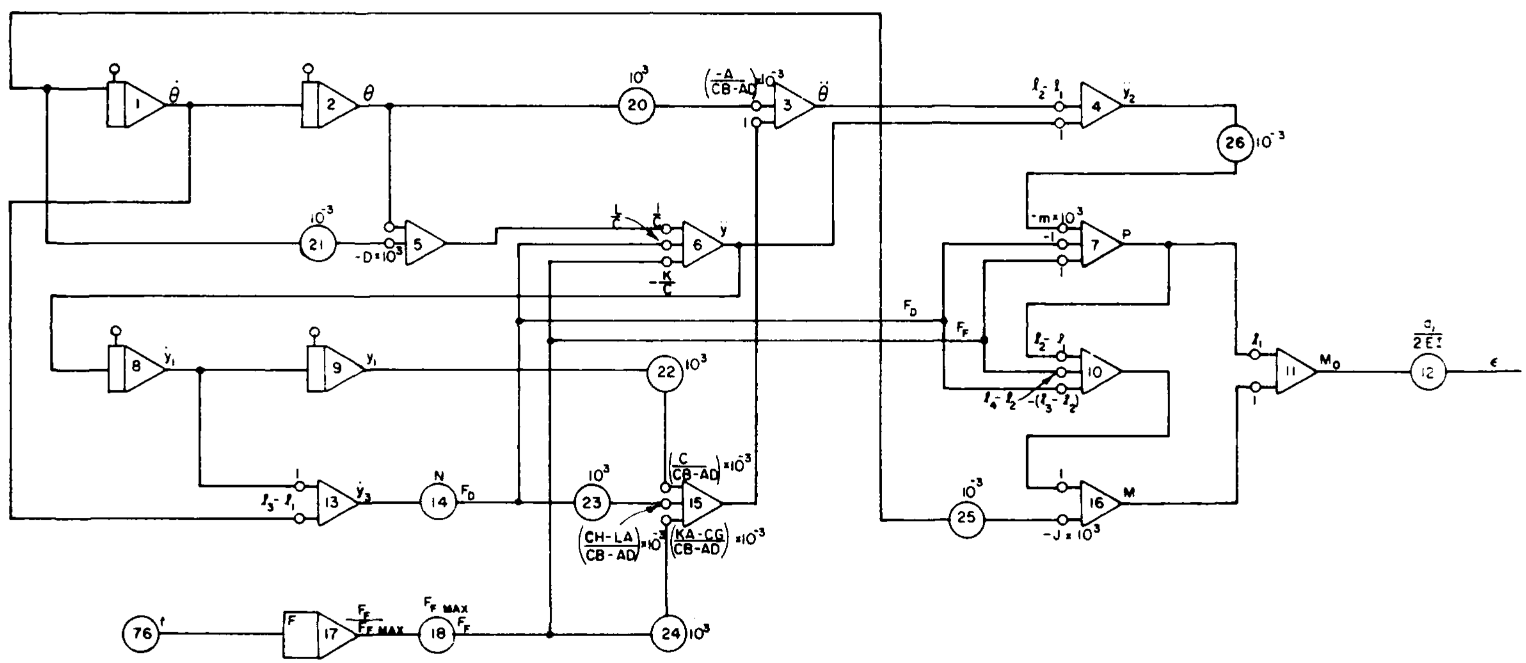


Figure 11: Complete CSMP Block Diagram

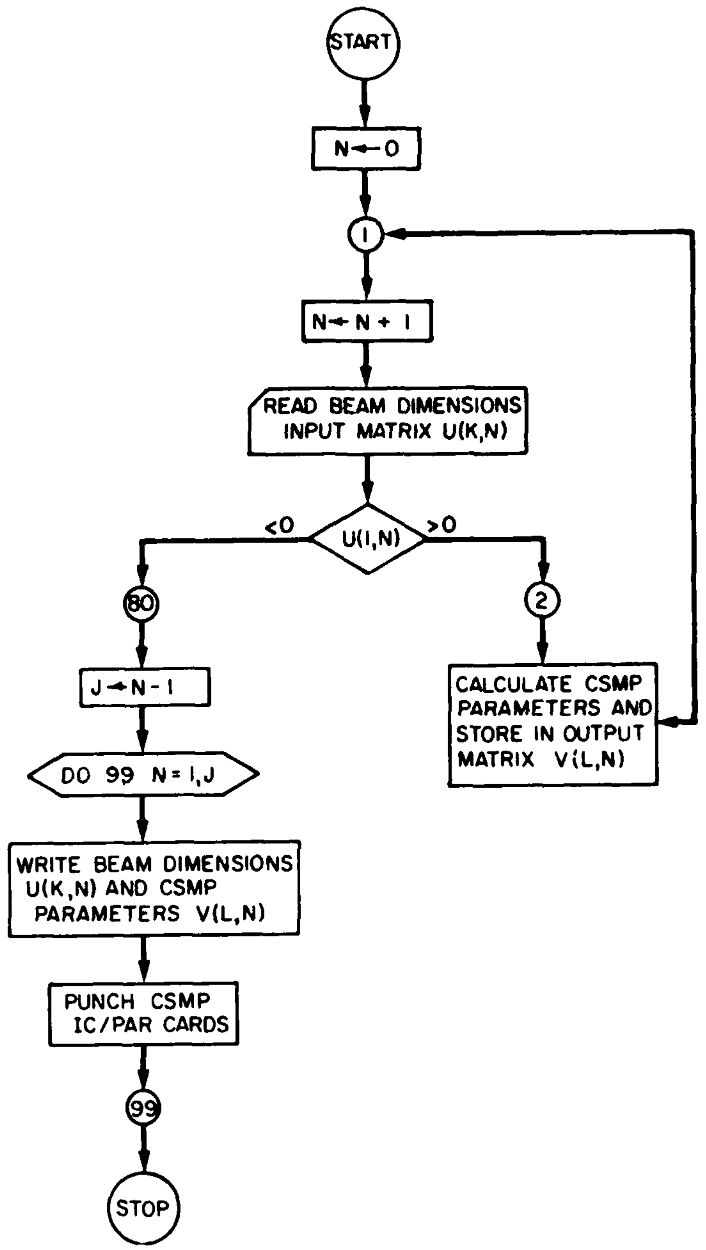


FIG. 12: FORTRAN PROGRAM FLOWCHART

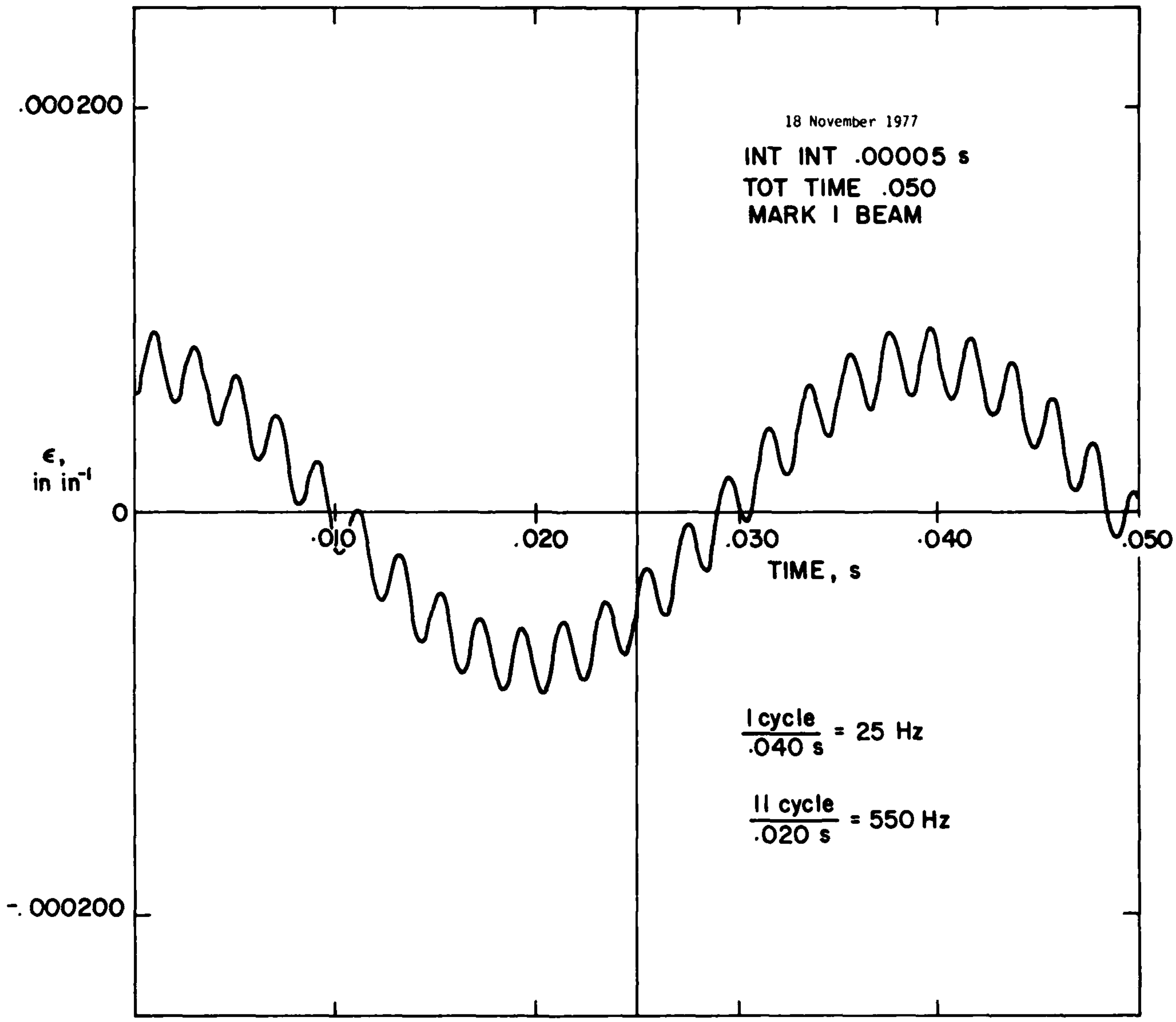


FIG. 13: STRAIN VS TIME - RUN I

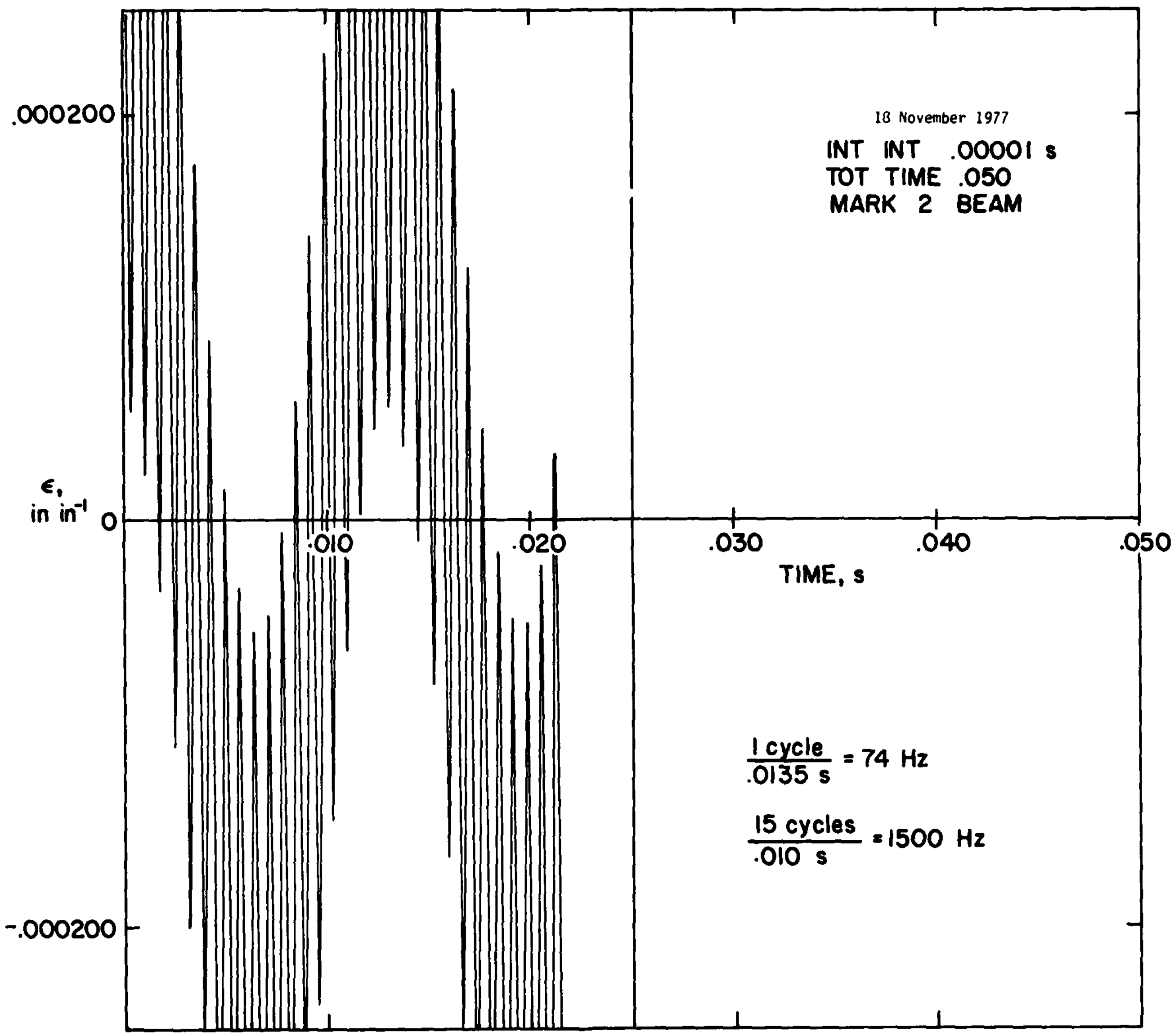


FIG. 14: STRAIN VS TIME - RUN 2

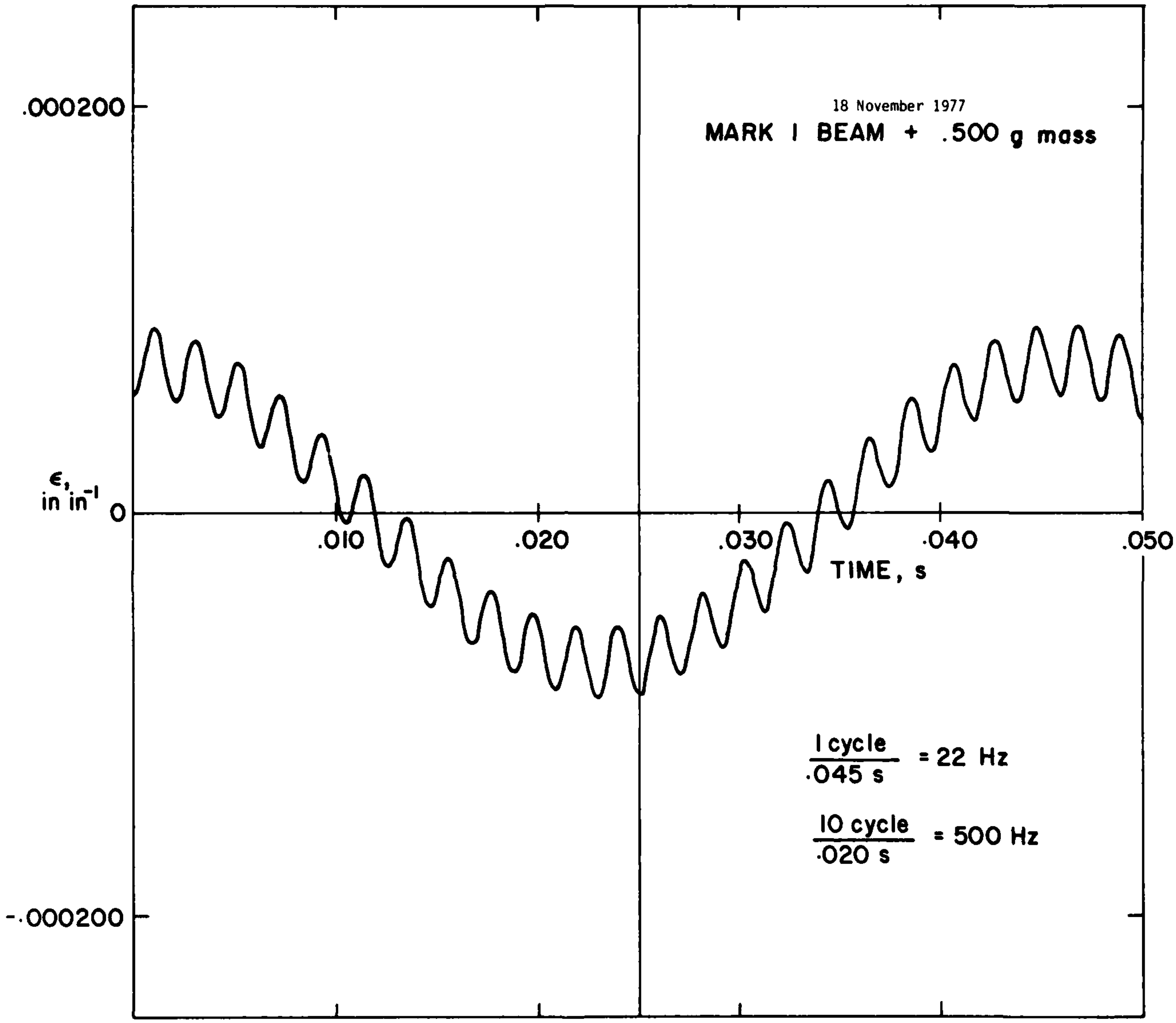


FIG. 15: STRAIN VS TIME - RUN 3

APPENDIX A

CSMP PROGRAM LISTING

APPENDIX A - CSMP Program Listing

CONTINUOUS SYSTEM MODELING PROGRAM
A DIGITAL ANALOG SIMULATOR PROGRAM FOR THE IBM 1130

CONFIGURATION SPECIFICATION

OUTPUT NAME	BLOCK	TYPE	INPUT 1	INPUT 2	INPUT 3
THETA DOT	1	I	3	0	0
THETA	2	I	1	0	0
THETA DOT DOT	3	W	20	15	0
Y2 DOT DOT	4	W	3	6	0
	5	W	2	21	0
Y1 DOT DOT	6	W	5	14	18
	7	W	26	14	18
Y1 DOT	8	I	6	0	0
Y1	9	I	8	0	0
	10	W	7	18	14
	11	W	7	16	0
STRAIN	12	G	11	0	0
Y3 DOT	13	W	8	1	0
DAMPING FORCE	14	G	13	0	0
	15	W	22	23	24
	16	W	10	25	0
FORCING FUNCTION	17	F	76	0	0
PEAK FORCE	18	G	17	0	0
	20	G	2	0	0
	21	G	3	0	0
	22	G	9	0	0
	23	G	14	0	0
	24	G	18	0	0
	25	G	3	0	0
	26	G	4	0	0

INITIAL CONDITIONS AND PARAMETERS

IC/PAR NAME	BLOCK	IC/PAR1	PAR2	PAR3
THETA	2	0.011970	0.000000	0.000000
	3	-19112.937553	1.000000	0.000000
	4	0.825000	1.000000	0.000000
	5	1.000000	0.003253	0.000000
	6	-315561.813232	-102316.484558	137598.031677
	7	-0.012597	-1.000000	1.000000
Y1	9	0.005250	0.000000	0.000000
	10	0.825000	0.825000	-0.325000
	11	0.600000	1.000000	0.000000
	12	0.003720	0.000000	0.000000
	13	1.000000	1.150000	0.000000
	15	58509.007904	-113.380005	287.830017
	16	1.000000	-0.002858	0.000000
FORCING FUNCTION	17	0.500000	0.000000	0.000000
	20	1000.000123	0.000000	0.000000
	21	0.001000	0.000000	0.000000
	22	1000.000123	0.000000	0.000000
	23	1000.000123	0.000000	0.000000
	24	1000.000123	0.000000	0.000000
	25	0.001000	0.000000	0.000000
	26	0.001000	0.000000	0.000000

FUNCTION GENERATOR SPECIFICATIONS

17	0.0000	1.0000	0.7246	0.5252
	0.3806	0.2758	0.2000	0.1448
	0.1050	0.0760	0.0552	

```
(          ) INTEGRATION INTERVAL
 .00005

(          ) TOTAL TIME
  .100

(   ) BLOCK FOR Y-AXIS   (           ) MINIMUM VALUE     (           ) MAXIMUM VALUE
 12                       -.000250                        .000250

(   ) BLOCK FOR X-AXIS   (           ) MINIMUM VALUE     (           ) MAXIMUM VALUE
 76                          0.                             0.100
 PREPARE PLOTTER AND PRESS START
 SET PEN ABOUT ONE INCH FROM RIGHT MARGIN

(          ) PRINT INTERVAL
  .005
        TIME      OUTPUT(   )   OUTPUT(   )   OUTPUT(   )   OUTPUT(   )   OUTPUT(   )
                          2             9             7            16            12
 OUTPUT ON LINE PRINTER
RUN TERMINATED BY SWITCH 0

AFTER SELECTING DESIRED OPTION PRESS START
```

APPENDIX B

FORTRAN PROGRAM LISTING

APPENDIX B - Fortran Program Listing

```
PAGE   1

// JOB T

LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        0301         0301        0000

V2 M10   ACTUAL 16K  CONFIG 16K

// FOR
*ONE WORD INTEGERS
*LIST ALL
*IOCS(CARD)
*IOCS(1132 PRINTER)
C
C     CALCULATION OF PARAMETERS FOR CSMP VIBRATING CANTILEVER PROGRAM
C             DRESMES    D.A.BAYLY
C
      DIMENSION U(11,5), V(17,5), EMM(5)
      N= 0
    1 N= N + 1
      READ(2,100) U(1,N),U(2,N),U(3,N),U(4,N),U(5,N),U(6,N),U(7,N),U(8,N
     1),U(9,N),U(10,N),U(11,N)
  100 FORMAT(F5.3,2X,F5.3,2X,F5.3,2X,F5.3,2X,F5.3,2X,F5.3,2X,F5.3,2X,F5.
     13,2X,F5.3,2X,F9.0,2X,F5.3)
      IF( U(1,N) ) 80,80,2
    2 FL1 = U(1,N)
      FL2 = U(2,N)
      FL3 = U(3,N)
      FL4 = U(4,N)
      B1 = U(5,N)
      B2 = U(6,N)
      D1 = U(7,N)
```

```
D2 = U(8,N)
W = U(9,N)
E = U(10,N)
ADMAS  = U(11,N)
EM = .002588*W*B2*D2*(EL4 - EL1)
REALJ = .08333*EM*(((EL4 - EL1)**2.) + D2*D2 )
REALI = .08333*B1*(D1**3.)
EM = EM + ADMAS * .0000057
A = ((-EM*EL1*EL1)*(EL2 - EL1/3.)) / (2.*E*REALI)
B= (-EL1*EL1/(E*REALI)) * (EM*(EL1*EL1/6. - .6667*EL1*EL2 +  .50*E
1L2*EL2) + .5*REALJ  )
C = (-EM*EL1/(E*REALI))*(EL2 - .500*EL1)
D = (-EL1/(E*REALI)) * ( EM*(EL2*EL2 - 1.5*EL1*EL2 + .5*EL1*EL1)
1+ REALJ )
G =(.500*EL1*EL1/(E*REALI))*(EL4 - EL1/3.)

PAGE   2

H =(.500*EL1*EL1/(E*REALI))*(EL3 - EL1/3.)
REALK =(EL1/(E*REALI)) * (EL4 - EL1/2.)
REALL =(EL1/(E*REALI)) * (EL3 - EL1/2.)
P1B3 =( -A /( C*B - A*D))*.001
P1B4 =  EL2 - EL1
P2B5 = -D*1000.
P1B6 = 1./C
P2B6 = REALL/C
P3B6 = -REALK/C
P1B7 = -EM*1000.
P1B10 = EL2 - EL1
P2B10 = EL4 - EL2
P3B10 =  EL2 - EL3
P1B11 =  EL1
P1B12 = .5*D1/(E*REALI)
P2B13 =  EL3 - EL1
```

```
      P1B15 = (C / (C*B - A*D))*.001
      P2B15 =((C*H - RFALL*A)/(C*B - A*D))*.001
      P3B15 =((RFALK*A - C*G)/(C*B - A*D ))*.001
      P2B16 = -RFALJ*1000.
      EMM(N) = EM*175400. - ADMAS
      V(1,N) = P1B3
      V(2,N) = P1B4
      V(3,N) = P2B5
      V(4,N) = P1B6
      V(5,N) = P2B6
      V(6,N) = P3B6
      V(7,N) = P1B7
      V(8,N) = P1B10
      V( 9,N) = P2B10
      V(10,N) = P3B10
      V(11,N) = P1B11
      V(12,N) = P1B12
      V(13,N) = P2B13
      V(14,N) = P1B15
      V(15,N) = P2B15
      V(16,N) = P3B15
      V(17,N) = P2B16
      GO TO 1
   80 J = N-1
      DO 99 N=1,J
      WRITE(3,101)
  101 FORMAT(11X,'EL1',4X,'EL2',4X,'EL3',4X,'EL4',4X,'B1',5X,'B2',5X,'D1
     1',5X,'D2',5X,'W',7X,'E',8X,'ADMAS ',/,11X,'IN',5X,'IN',5X,'IN',5X,
     2'IN',5X,'IN',5X,'IN',5X,'IN',5X,'IN',4X,'PCI',5X,'PSI',9X,'G',/)
      WRITE(3,102) U(1,N),U(2,N),U(3,N),U(4,N),U(5,N),U(6,N),U(7,N),U(8,
```

PAGE 3

```
      1N),U(9,N),U(10,N),U(11,N)
  102 FORMAT(10X,F5.3,2X,F5.3,2X,F5.3,2X,F5.3,2X,F5.3,2X,F5.3,2X,F5.3,2X
     1,F5.3,2X,F5.3,2X,F9.0,2X,F5.3,///)
      WRITE(3,140) EMY(N)
  140 FORMAT(30X,'BEAM MASS = ',F5.3,' G',//,14X,'BLOCK NO.',2X,'I/C PAR
     1 1',6X,'I/C PAR 2',6X,'I/C PAR 3',/)
      WRITE(3,203) V(1,N)
  203 FORMAT(19X,'3',5X,F10.3,5X,'1.0')
      WRITE(3,204) V(2,N)
      WRITE(3,205) V(3,N)
      WRITE(3,206) V(4,N),V(5,N),V(6,N)
      WRITE(3,207) V(7,N)
      WRITE(3,210) V(8,N),V(9,N),V(10,N)
      WRITE(3,211) V(11,N)
      WRITE(3,212) V(12,N)
      WRITE(3,213) V(13,N)
      WRITE(3,215) V(14,N),V(15,N),V(16,N)
  215 FORMAT(18X,'15',5X,F10.2,5X,F10.2,5X,F10.2)
      WRITE(3,216) V(17,N)
      WRITE(3,220)
  220 FORMAT(////////)
      WRITE(2,203) V(1,N)
      WRITE(2,204) V(2,N)
  204 FORMAT(19X,'4',5X,F10.3,5X,'1.0')
      WRITE(2,205) V(3,N)
  205 FORMAT(19X,'5',5X,'1.0',12X,F10.8)
      WRITE(2,206) V(4,N),V(5,N),V(6,N)
  206 FORMAT(19X,'6',5X,F10.2,5X,F10.2,5X,F10.2)
      WRITE(2,207) V(7,N)
  207 FORMAT(19X,'7',5X,F10.7,5X,'-1.',12X,'1.')
      WRITE(2,210) V(8,N),V(9,N),V(10,N)
  210 FORMAT(18X,'10',5X,F10.3,5X,F10.3,5X,F10.3)
      WRITE(2,211) V(11,N)
  211 FORMAT(18X,'11',5X,F10.3,5X,'1.0')
```

```
      WRITE(2,212) V(12,N)
  212 FORMAT(18X,'12',5X,F10.5)
      WRITE(2,213) V(13,N)
  213 FORMAT(18X,'13',5X,'1.0',12X,F10.3)
      WRITE(2,215) V(14,N),V(15,N),V(16,N)
   99 WRITE(2,216) V(17,N)
  216 FORMAT(18X,'16',5X,'1.0',12X, F10.7)
      CALL EXIT
      END

// XEQ
```

FL1 IN	FL2 IN	FL3 IN	FL4 IN	B1 IN	B2 IN	D1 IN	D2 IN	W PCI	E PSI	ADMAS G
0.750	1.875	2.500	3.000	0.250	0.375	0.010	0.010	0.295	16100002.	0.000

BEAM MASS = 1.129 G

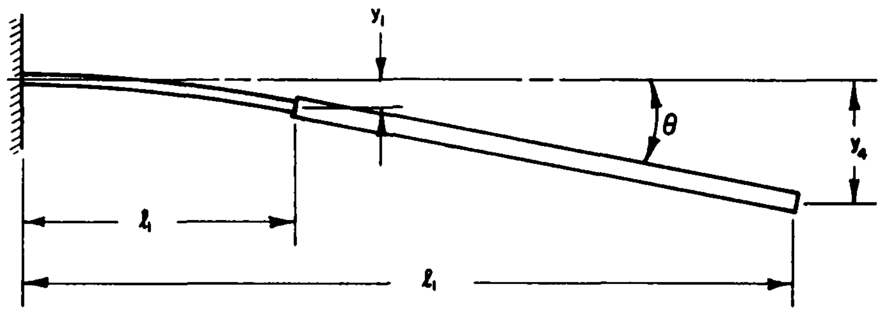
BLOCK NO.	I/C PAR 1	I/C PAR 2	I/C PAR 3
3	-2133.610	1.0	
4	1.125	1.0	
5	1.0	0.03038400	
6	-46282.39	-219921.59	271667.87
7	-0.0064416	-1.	1.
10	1.125	1.125	-0.625
11	0.750	1.0	
12	0.01490		
13	1.0	1.750	
15	5251.96	-229.37	412.87
16	1.0	-0.0027175	

APPENDIX C

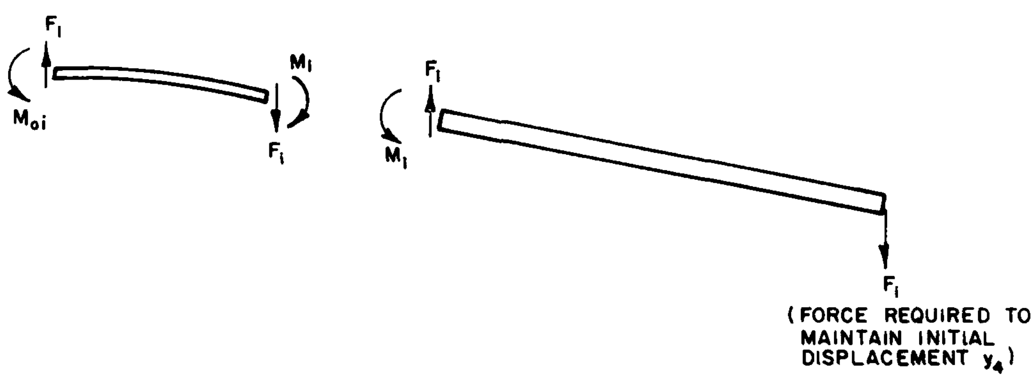
CALCULATION OF INITIAL y_1 AND θ FROM INITIAL y_4

APPENDIX C - Calculation of initial y_1 and θ from initial y_4

DIMENSIONS



FREE BODY DIAGRAMS



Summing moments on the rigid portion of the beam,

$$M_i = F_i(\ell_4-\ell_1) \quad . \tag{33}$$

The deflection and rotation at the end of the elastic portion are

$$y_1 = \frac{F_i\ \ell_1^{\ 3}}{3EI} + \frac{F_i(\ell_4-\ell_1)\ell_1^{\ 2}}{2\ EI}$$

$$= \frac{F_i}{EI}\left[\frac{\ell_1^{\ 3}}{3} + \frac{(\ell_4-\ell_1)\ell_1^{\ 2}}{2}\right] \tag{34}$$

and

$$\theta = \frac{F_i(\ell_4-\ell_1)}{EI} + \frac{F_i\ \ell_1^{\ 2}}{2EI}$$

$$= \frac{F_i}{EI}\left[(\ell_4-\ell_1) + \frac{\ell_1^{\ 2}}{2}\right] \quad . \tag{35}$$

Assuming that the rotation θ is small,

$$y_1 = y_4 - (\ell_4-\ell_1)\theta \quad . \tag{36}$$

Substituting equation (36) in (34)

$$y_4 - (\ell_4-\ell_1)\theta = \frac{F_i}{EI}\left[\frac{\ell_1^{\ 3}}{3} + \frac{(\ell_4-\ell_1)\ell_1^{\ 2}}{2}\right] . \tag{37}$$

Rearranging (37)

$$\theta = \frac{y_4}{(\ell_4-\ell_1)} - \frac{F_i}{EI(\ell_4-\ell_1)}\left[\frac{\ell_1^{\ 3}}{3} + \frac{(\ell_4-\ell_1)\ell_1^{\ 2}}{2}\right] . \tag{38}$$

Equating the right-hand side of (35) and (38)

$$\frac{F_i}{EI}\left[(\ell_4-\ell_1)\ell_1 + \frac{\ell_1^{\ 2}}{2}\right]$$

$$= \frac{y_4}{(\ell_4-\ell_1)} - \frac{F_i}{EI(\ell_4-\ell_1)}\left[\frac{\ell_1^{\ 3}}{3} + \frac{(\ell_4-\ell_1)\ell_1^{\ 2}}{2}\right] . \tag{39}$$

Rearranging (39)

$$F_i = \frac{EI\ y_4}{(\ell_4-\ell_1)} \times \frac{1}{\left\{(\ell_4-\ell_1)\ell_1 + \frac{\ell_1{}^2}{2} + \frac{1}{(\ell_4-\ell_1)}\left[\frac{\ell_1{}^3}{3} + \frac{(\ell_4-\ell_1)\ell_1{}^2}{2}\right]\right\}} \cdot \qquad (40)$$

Substituting for F_i in equations (34) and (35)

$$y_1 = y_4 \times \frac{\left[\frac{\ell_1{}^3}{3} + \frac{(\ell_4-\ell_1)\ell_1{}^2}{2}\right]}{(\ell_4-\ell_1)\left\{(\ell_4-\ell_1)\ell_1 + \frac{\ell_1{}^2}{2} + \frac{1}{(\ell_4-\ell_1)}\left[\frac{\ell_1{}^3}{3} + \frac{(\ell_4-\ell_1)\ell_1{}^2}{2}\right]\right\}} \cdot \qquad (41)$$

$$\theta = y_4 \times \frac{\left[(\ell_4-\ell_1)\ell_1 + \frac{\ell_1{}^2}{2}\right]}{(\ell_4-\ell_1)\left\{(\ell_4-\ell_1)\ell_1 + \frac{\ell_1{}^2}{2} + \frac{1}{(\ell_4-\ell_1)}\ \frac{\ell_1{}^2}{3} + \left[\frac{(\ell_4-\ell_1)\ell_1{}^2}{2}\right]\right\}} \cdot \qquad (42)$$

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A cantilever beam force transducer was modelled as a massless elastic section at the base with the remaining section of the beam rigid and having mass. Computer programs were written to simulate free or forced, damped or undamped vibrations of the beam. Good agreement was found between predicted and experimental frequencies of undamped free vibration for two different beams. After further verification, the computer programs can be used to determine beam configurations, viscous damping factors, and loading rates which will reduce unwanted oscillations of the transducer element.

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Cantilever
Beam
Transducer
Force
Vibrations
Damped
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